THE CONTROVERSY OF TEACHING CALCULUS
IN HIGH SCHOOL

by

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CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

I. INTRODUCTION

During the past decade, the desirability of including calculus in the high school curriculum has been discussed at some length. The study of calculus in high school, contrary to popular thought, is not a new idea as will be shown in the historical review of the literature, but until recently it was largely an academic question as the number of students exposed to such a study was very small. Traditionally though, the study of calculus has been omitted from the American high school curriculum. It has been the opinion of the majority of mathematicians and educators that the average American high school student did not have the ability or the background to understand the basic concepts of calculus and therefore felt that the teaching of calculus should be left to the colleges and universities [31, 1][1].

However, after the launching of the Russian Sputnik in 1957, a reversal of opinion on this matter seemed to have occurred. The American nation focused its training of young people to meet the rapidly expanding demands for engineers, scientists, and mathematicians. Curriculum changes began to take place upon recommendations by the Commission of Mathematics of

[1] The first coordinate of the ordered pair refers to the number of the article as listed in the bibliography. The second coordinate refers to the page number of the article from which the quotation or reference was taken.

the College Entrance Examination Board, the School Mathematics Study Group, the University of Illinois Committee on School Mathematics, and other similar groups. Subject matter has been reorganized and drill for drill's sake has been abandoned. The seventh and eighth grade mathematics contain concepts of algebra and geometry in order to prepare the student for high school mathematics. In many schools, algebra is offered in the eighth grade with geometry following in the ninth grade. We see that solid geometry and trigonometry are losing their place as a full year course and are being included in plane geometry and algebra II. The emphasis in trigonometry has shifted from the logarithmic solution of right triangles to trigonometric functions and their properties. These and other factors have operated to produce at least one semester, and in many cases a full year of free time at the twelfth grade level.

This free time at the twelfth grade level has presented a challenging problem in curriculum development. Filling the resulting gap between trigonometry in the eleventh grade and calculus on the freshman college level is a problem confronting teachers, administrators, and textbook writers.

One view concerning content for twelfth grade is expressed by Albert A. Blank.

> Calculus is a natural cap to the high school curriculum. It reinforces by utilization all the concepts and techniques learned earlier. In its manipulative and problem-solving aspects the calculus is entirely in the spirit of secondary mathematics. No alternative to the calculus is superior for opening so many avenues to higher mathematics, to physical sciences and technology, and even to the biological, management, and social sciences [8, 13].

Naturally, Blank's view is not shared by all individuals or curriculum groups. Some of the criticism of such a course is no doubt motivated by the belief that other topics such as probability, statistics, elementary functions, linear algebra and matrices, analytic geometry, modern algebra, and computer programming would be more profitable for the student. "The Commission on Mathematics of the College Entrance Examination Board feels that 'Calculus is a college level subject' and 'a reasonably immediate goal for most high schools is a strong college-preparatory mathematics curriculum that will have students ready to begin calculus when they enter college' [12, 3]."

II. PURPOSE OF THE PAPER

The purpose of this report is to review the literature concerning the controversy of teaching high school calculus and to collect the literature into a unified whole enabling one to get a quick picture of the work that has been done on this problem. A report of this kind will enable high school teachers and administrators to (1) gain insight into the problems they face in their school, (2) determine under what conditions calculus can be taught successfully in high school, (3) analyze the framework under which students may take calculus in high school, and (4) aid them in determining the content of the fourth year mathematics course for their high school curriculum.

CHAPTER II

REVIEW OF THE LITERATURE

I. HIGH SCHOOL CALCULUS PRIOR TO 1900

The American high school during the Colonial period offered mathematics courses that dealt primarily with integral numbers and the basic concepts of fractions and proportions. Thus the topics of calculus were not taught during this period of American History because of the lack of the basic skills and concepts necessary for such a course. It must be remembered that the work of Cauchy (1789-1857) and Weierstrass (1815-1897) was not until the nineteenth century [12, 8].

During the nineteenth century American schools began to teach a few of the concepts of higher mathematics rather than stressing the more difficult manipulations of arithmetic. Central High School in Philadelphia is remembered for having offered a course in the theory of limits in 1845 [31, 6]. A few other schools slowly followed suit. A Baltimore high school offered a similar course in 1851[31, 6]. Meanwhile, in the North Central states, high schools in Racine, Wisconsin and Detroit, Michigan offered a course in analytic geometry in 1858 and 1859, respectively. A high school in St. Louis, Missouri offered a one semester course in analytic geometry in 1867 [31, 6]. Thus a few of the prominent mathematics educators in this country during that period recognized that high school students were capable of studying and understanding more abstract concepts than had previously

been studied in high school. However by the turn of the century most of these schools ceased to offer calculus in any form [31, 7].

The American high school offered a practical, terminal program like that of the early academies with courses in surveying, navigation, and engineering. They also appeared to be anxious to compete with the colleges. For either role, calculus seemed to be a logical part of the curriculum [31, 7].

The rapid disappearance of the calculus from most high schools may have accompanied a reappraisal of the role of the institution with regard to training the future scientists and engineers. The lack of vigor and intelligibility in the subject as treated by the textbooks of that period may have been another factor.

II. HIGH SCHOOL CALCULUS FROM 1900 TO 1930

The movement to include calculus in the high school curriculum probably received most of its stimulus from Europe. During the 1910's in a bulletin concerning curriculums abroad, calculus was shown to be offered in the twelfth year of secondary schools in Austria, Belgium, Denmark, England, France, Germany, Roumania, Sweden, and Switzerland; and in the tenth year in Russia. In Holland and Hungary, analytic geometry was taught but not the calculus. The entry for the United States merely states: "Analytic Geometry is seldom taught in secondary schools [31, 13-14]."

Two Europeans prominent in this movement were Felix Klein of Germany and John Perry of England. Mr. Klein advocated a calculus course for the

students who were attending his "real Gymnasium", the German secondary school. He believed that one could slowly develop the concept of slope and area to pupils fourteen and fifteen years old, provided that it was made clear to the pupil that he was dealing with simple things anyone could understnad [28, 12].

Professor John Perry, of the Royal College of Science in London, was against the teaching of the traditional mathematics courses because he felt they would be of little practical value to his students in later life, whereas, he felt the applications of the concepts of calculus were of sufficient importance to include such a course in the high school mathematics curriculum and that such a course would improve the attitude of the students toward mathematics.

There are a number of reasons why American educators did not follow the European educational trends. Perhaps the foremost reason is that our system of education is not as centralized as theirs. Each state has its own system of education with its own separate goals and purposes of education.

Perhaps two of America's first educators to become instrumental in the movement to include a calculus course in the high school curriculum were Professor E. H. Moore (1902) of the University of Chicago and David Eugene Smith (1908) of Teachers College, Columbia University. They both emphasized the practical side of mathematics, and problems related to physics.

It is interesting to note that in the 1920's Noah Rosenberger undertook a study to determine the attitudes of educators concerning the controversy of the instruction of high school calculus. He cited the major reason for

justifying the inclusion of calculus in high school as its importance in engineering, physics, and mechanics. Some mathematicians felt that the study of calculus in the twelfth-grade would provide a good link between the previous school work in mathematics and the more pure forms of abstract mathematics. Mr. Rosenberger also indicated that the principal hindrance to this movement at that time was the lack of a good high school textbook and the lack of qualified teachers who were capable of teaching a calculus course at the high school level. It is interesting to note that these same arguments are cited today [30, 153-5].

Another viewpoint was expressed by Miss Susie Farmer in a 1927 article of The Mathematics Teacher. She stressed the social and cultural advantages of the high school calculus course and advocated its teaching for the pure truth and beauty of calculus rather than for the practical aspects of calculus. Miss Farmer's views are expressed in the following quotation:

> However, higher mathematics has contributed so much of practical and cultural value to civilization that the very existence of our social fabric depends upon it. Its socializing value rests in the pupil's proper attitude toward the specialist upon whose work our social life depends. Even though calculus should not be of any practical value to any particular child, he should not be denied the privilege of enjoying its pursuit but should have as good a right to develop that ability as any other ability [18, 186-7].

There were of course many educators who opposed the teaching of calculus to high school students. Professor W. F. Babcock at Woodmere Academy, Woodmere, New York, summarised a 1927 survey of mathematics department heads of colleges in the eastern United States by saying: "I doubt if a student younger than eighteen years even understands analytic reasoning.

I doubt the wisdom of beginning the calculus then [4, 479]."

III. HIGH SCHOOL CALCULUS FROM 1930 TO 1955

During the 1930's the value of the old mental discipline theory of education was seriously doubted and moving to the foreground was what is called the Progressive Movement in education. The Progressive Movement, stimulated by the economic depression of the time, emphasized practical mathematics such as business and consumer mathematics. This movement tended to hinder the introduction of the more abstract concepts of calculus into the high school curriculum and in many cases the entire high school mathematics curriculum was almost forgotten.

The inadequacy of mathematical preparation of army draftees during World War II pointed out the poor training students of secondary schools were obtaining. The rapid development of the physical and mathematical sciences during and after the war created needs for highly trained teachers. As a result, according to Donald Tillotson, three major interests developed in the post war period:

1) the improvement of the minimum program in mathematics for all American youth,
2) the development of the abilities of highly gifted students,
3) the modification of the curriculum and teaching methods to produce a "modern" mathematics program [31, 24-5].

Committees such as the Commission on Post-War plans formed by the National Council of Teachers of Mathematics presented suggestions for improving mathematical instruction in secondary schools. These proposals dealt with "the place of mathematics in the curriculum of grades one through

fourteen; the prevalent view of arithmetic; the development of meanings; readiness as a function of experience; unifying themes; multiple tracks for large high schools; preparation of teachers; and basic concepts for organization of subject matter[12, 27]."

Very little progress was made in this movement until the 1950's. During this time mathematics was poorly appreciated; it was difficult to justify, and often misfit leaders and football coaches taught it [17, 515].

In 1952, the College Entrance Examination Board appointed a committee known as the Commission on Mathematics whose reports have influenced various writing groups and state and local curriculum groups throughout the country. The commission concluded that "calculus is a college-level subject" and high schools should have their students prepared to start calculus when they enter college[12, 30]. The work of the Commission on Mathematics has resulted in what is known today as the Advanced Placement Program in Mathematics.

In 1955, the commission recommended a mathematics curriculum for academically advanced college bound high school students. The twelfth-grade course consisted of an introduction to calculus which included analytic geometry, differential calculus with applications, and integral calculus with applications [28, 24-5].

There have been many proposals for changes in curriculum made in the past decade. The material produced by the School Mathematics Study Group (SMSG) represents the largest united effort for improvement in the history of mathematics education [12, 32]. Their sample textbooks represent the

combined thinking of psychologists, testmakers, mathematicians, biologists, and high school teachers.

IV. HIGH SCHOOL CALCULUS FROM 1955 TO THE PRESENT

During the past decade the thought of teaching calculus in high school has been stimulated by what may be considered as two major revolutions in the teaching of mathematics. The first revolution in mathematics was stimulated by the successful firing of the first earth satellite, Sputnik I, by the Russians. Although this revolution had been brewing since before World War II, it had not entered its active phase and no real progress had been made.

After the firing of Sputnik I, an increased emphasis was put upon the teaching of science and mathematics in our public schools. Federal funds became readily available to schools and students of science and mathematics. Mathematical committees such as SMSG were organized for the purpose of improving the mathematics education in our schools. New goals for the teaching of mathematics were established. The primary goal is to make every student a competent user of the mathematics he will use in his life and career. More specifically, the School Mathematics Study Group (SMSG) is seeking to provide every student with greater facility in the basic skills of arithmetic, algebra, and geometry; to give him a better understanding of mathematics for further study; to give him greater facility in the conversion of a practical problem into a form suitable for mathematical analysis; and to eliminate the general fear of quantitative thinking in a mathematical language [5, 195-6].

The first versions of the SMSG materials were available in the fall of 1959 and were generally released in form for use in 1960. The nature of the SMSG materials was such that it was generally aimed at the college-capable students.

A second revolution in mathematics came out of the Cambridge Conference of 1963. The Conference was composed of a group of twenty-five mathematicians and other scientists selected from the most prestigious institutions in the country. The goal of the Conference was to forecast the shape and content of the mathematics curriculum of the year 1990. Their findings were published in a report called "Goals for School Mathematics," now usually referred to as the Cambridge Report. The Cambridge Conference called for a complete reconstruction of the mathematics curriculum, beginning in kindergarten and proceeding through the high school to the colleges. The main conclusion of the report is stated in these words:

> A student who has worked through the full thirteen years of mathematics in grades K to 12 should have a level of training comparable to three years of top-level college training today; that is, we shall expect him to have the equivalent of two years of calculus, and one semester each of modern algebra and probability theory [1, 210].

This is to be accomplished by improving the mathematical program in the elementary school to the point where it includes much of the present first two years of the high school. These goals cannot be achieved until there has been a complete revision in the training of mathematics teachers in the elementary and secondary schools.

One wonders if the current reform in the mathematics curricula is "a passing phase or progress." W. Eugene Ferguson, Newton High School, Newtonville, Massachusetts, answers emphatically, "It is progress," but hastens to add that in the development of the new curricula some of the individual experiments have been passing phases [21, 143]. Many things have been tried that did not work out and those have been discarded. These failures have only spurred educators on to find better methods of treating the very difficult parts of the new programs.

New concepts are finding their places in the secondary school system, although the old traditional content is still the basis of all the new programs. Mr. Ferguson states that for many teachers the new approach to the old familiar content is the most difficult aspect of the reform movement. After the teachers become acquainted with the new approach and understand its goals they too become excited about the fresh approach to the mathematical content and the discovery method of teaching.

The current reforms have probably had the greatest effect on teachers. They have sent the teachers to in-service training programs to update their mathematics. Teachers report they have never worked so hard in all their lives, but they also say they enjoy it more. They discover that the new programs of mathematics are not easy, but that they present more of a challenge to students and teachers alike and they are more interesting.

Mr. Ferguson's article in The Mathematics Teacher indicates that the new mathematics programs have their effect on students too. Students are finding mathematics to be interesting and exciting when they approach these

new programs with an open mind. One student made this comment, "I surely enjoyed mathematics even though I didn't make very good marks in it [21, 145]." Parents report that their children are liking mathematics and they want to know what is producing this change in attitude. Often before parents told their children that they hated mathematics when they took it in school. A negative attitude like this, of course, could not inspire children to achieve their maximum potentials. Students are experiencing the thrill of discovering mathematical principles and of seeing the structure, order, and the beauty of mathematics. Even the poor students are liking mathematics more. Mr. Ferguson believes that one of the chief problems in teaching mathematics is motivation of the student, and these new programs under the direction of a well qualified teacher seem to be solving that problem.

Even though new methods of teaching mathematics are readily taking hold, the question of whether calculus should be offered in high school still remains a strongly debated issue.

CHAPTER III

THE CASE FOR CALCULUS

We are just emerging from a decade of reform in mathematics education in which the calculus has been moving into the high schools at an alarming rate. The Advanced Placement Program was being proposed at the same time that our schools were being criticized for not properly preparing young people to fill the ranks of students in technological areas where critical shortages existed. Thus the time was ripe for the teaching of calculus in high school to take hold quickly. Rarely has a program been received as widely and quickly as this program for teaching calculus in high school. It is now time to look at our problems in perspective and ask what we desire to accomplish in the high school mathematics curriculum. Is calculus really a good course to offer in high school?

Frank B. Allen, president of National Council of Teachers of Mathematics (NCTM), reported in 1961 that because of the advanced placement courses in high schools, 9 percent of the M. I. T. freshmen entered with advanced credit for the first semester of calculus and 11 percent more got credit for a full year [2, 200-2]. Cal Tech gave 20 percent of their freshmen permission to skip the first half of the calculus course. Thus Allen says that high schools are just beginning to do a better job of preparing their students for college mathematics. In view of statements like these it seems that high schools which do not offer calculus to their students are not meeting their responsibilities.

I. BACKGROUND FOR THE HIGH SCHOOL CALCULUS STUDENT

When discussing the case for high school calculus it is assumed the student has successfully completed elementary algebra, geometry, intermediate algebra, and an advanced mathematics course. To give a complete description of the mathematical background of the high school calculus student, the series of high school textbooks published by the Houghton Mifflin Company is described.

Elementary Algebra. The goal of Houghton Mifflin's book 1, Modern Algebra, is to help the student to:

1) understand some of the basic structure of algebra (the real number system);
2) recognize the techniques of algebra as reflections of this structure;
3) acquire facility in applying algebraic concepts and skills;
4) perceive the role of deductive reasoning in algebra;
5) appreciate the need for precision of language [16, 2].

The elementary algebra is taught from the standpoint of structure using the properties of a field and is developed in the following way. The course begins with the four fundamental operations with rational numbers. These operations are then applied to the solutions of equations in one variable. This is followed by the axioms of inequality, inequalities in one variable, and applications to problem solving. The indirect method of proof is introduced here. The solution sets and graphs of open sentences in two variables are studied. Also included are methods of solving systems of open sentences, multiplication, factoring, and division of polynomials. This is followed by the four fundamental operations with rational expressions, direct, inverse, and combined variation, and irrational numbers which leads to the general

solution of a quadratic equation [16, 4].

If time would permit, additional topics such as computer programming or an introduction to the sine, cosine, and tangent functions could be included. A section on computer programming should have considerable vocational interest for students because of the increasing number of job opportunities in this field [16, 4].

Geometry. The authors of the Houghton Mifflin, Modern Geometry, believe that the students should be given a course in geometry which is of broader scope than that of the traditional plane geometry. Thus the important parts of solid geometry and coordinate geometry have been integrated with plane geometry. Such a course is not necessarily more difficult or demanding but it should help the student to:

1) understand the basic structure of geometry;
2) develop powers of spatial visualization while building his knowledge of the relationships among geometric elements;
3) grow in understanding of the deductive method and in appreciation of the need for precision of language;
4) use and strenghten his algebraic skills;
5) gain some knowledge of the methods of coordinate geometry and of the way in which algebra and geometry complement each other;
6) experience the stimulation and satisfaction that come from clear--and sometimes creative--thinking [23, 1-3].

The basic features of the text are these:

1) Sets are used throughout the book.
2) The Ruler and Protractor Postulates make arithmetic and algebra integral parts of the geometry course.
3) There is a summary of the properties of real numbers and order and betweenness relations are studied as well as inequalities.
4) The study of similarity before the study of circles permits an early presentation of the Pythagorean Theorem.

5) Several chapters are devoted to the basic ideas and methods of familiar theorems by coordinate methods, thereby strengthening the concept of algebraic proof.
6) After an early discussion in induction, the student is encouraged to make discoveries for himself and to test his conjectures. The student is led to an appreciation of the initial part played by induction in the development of a deductive system.
7) Informal proofs of early theorems avoid confusing the student with hairsplitting details he is not yet ready to appreciate. He can see that the theorems follow logically from a limited number of carefully-stated postulates. The necessity for a long list of postulates is avoided [23, 1-3].

Algebra II. Houghton Mifflin's book two, Modern Algebra and Trigonometry, has goals similar to the algebra book one: to clarify, simplify, unify, and broaden old ideas in mathematics by introducing new concepts that deepen understanding. The basic philosophy of this textbook is to help the student to:

1) understand algebra as a study of the structure of the systems of real and complex numbers;
2) recognize the techniques of algebra and trigonometry as reflections of this structure;
3) acquire facility in applying algebraic and trigonometric concepts and skills;
4) perceive the role of deductive reasoning in algebra and trigonometry;
5) appreciate the need for precision of language;
6) comprehend the function concepts and its importance in mathematics [15, 2].

The text discourages students from accepting vague and imprecise statements or seeking meaningless rules of what to do. Because the understanding of concepts and refinements of skills are both essential in mathematics, the text seeks to guide the student to discover mathematical principles as well as to provide ample exercise material to strengthen these principles and

to provide ample exercise material to strengthen these principles and basic skills.

Because the authors desire the student to know not only "What you do," and "How you do it," but also "Why you do it," the development of the course places considerable emphasis on the role of deductive reasoning in algebra. The students are led to see the need for a proof, to think in terms of an algebraic proof, and to construct proofs by themselves. The aim is to impress upon the student the fact that algebra, as an organized body of knowledge, can be developed from a few basic assumptions.

Chapters 1 through 5 cover the concepts of real numbers, skills involving operations with positive and negative numbers, solutions of linear equations and inequalities in two variables, solutions of verbal problems, properties of polynomials, and expressions of rational functions.

In chapters 6 through 9, the concepts of relation and function are developed, emphasizing linear and quadratic relations and functions, and the exponential and logarithmic functions over the system of real numbers. When the irrational numbers are developed, a short discussion of the Axiom of Completeness is included.

Chapters 10 through 12 discuss the trigonometric functions with a set of angles as the domain and the circular functions with the set of real numbers as the domain. Complex numbers are introduced in order to emphasize the interrelations among the concepts of trigonometry, vectors, and complex numbers.

In addition to these topics, chapters 13 through 16 include an introduction to matrices and determinants as well as the more traditional topics of algebra such as the binomial theorem, mathematical induction, permutations, combinations, and probability. It must be remembered that facility in algebraic manipulation cannot be replaced by an understanding of the structure of mathematics. The more manipulative facility the student acquires before starting calculus, the better [15, 1-5].

The Advanced Mathematics Course. The advanced mathematics course or pre-calculus course must provide a rich preparation for college courses in calculus, abstract algebra, and probability; and which can also serve as a terminal course for students who do not plan to continue their study of mathematics. The authors of the Houghton Mifflin series of high school mathematics textbooks state that this course should help the student to:

1) understand the role of logic in deductive systems of mathematics;
2) recognize that the manipulative techniques in a mathematical system are a reflection of the mathematical structure of that system;
3) acquire facility in applying mathematical techniques;
4) appreciate the breadth and depth of applications of mathematics;
5) prepare himself for modern courses in the calculus, abstract algebra, and probability;
6) perceive the unity of mathematics [14, 3].

Attention is given to the understanding of concepts, to the refinements of manipulative skills, and to the discovery of principles by the students. The textbook furnishes him with ample exercise material to strengthen his comprehension of these principles and to develop his appreciation of their usefulness. As in the other books in this series, the students are led to think in

terms of proof, to recognize the need for proof, and to devise proofs themselves. It is the aim to impress on every student that every branch of mathematics is an organized body of knowledge that can be derived from a few assumptions and undefined terms [14, 3].

In preparing this text, Modern Introductory Analysis, the authors have studied the recommendations of groups such as the Commission on Mathematics of the College Entrance Examination Board (CEEB), the School Mathematics Study Group (SMSG), and the various reports of the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America. These groups have sought to improve the mathematics programs in the high schools and colleges.

The major emphasis of Modern Introductory Analysis is the examination of the number systems of elementary mathematics and the study of the elementary functions. The basic content comprises the algebras of real numbers, vectors, complex numbers, and polynomials; analytic geometry based on vector algebra; polynomial, exponential, and logarithmic functions; the circular and trigonometric functions; and elementary probability functions.

Chapters 1 through 4 introduce the logic basic to deductive reasoning, the properties of the real number system with considerable emphasis on the role of proof by mathematical induction, the concept of limit of a sequence, and the algebra of vectors which is basic to the study of analytic geometry.

Chapter 5 uses the algebra of vectors to develop an analytic model of plane Euclidean geometry. This material is used in discussing the graphs of functions, the applications of the circular and trigonometric functions,

loci of second-degree equations, and an algebraic model of geometry in space.

Chapter 6 introduces polynomial functions and their zeros which leads to a discussion of complex numbers based on the algebra of vectors discussed in chapter 7.

Chapter 8 is designed to prepare the student for a mature course in calculus by developing good intuitive understanding of such concepts as the limit of a function, continuity, and derivative.

Chapter 9 presents a study of the composition of functions, the notion of positive integral exponents, and the notion of inverses which leads into logarithmic functions.

A rigorous study of the circular and trigonometric functions and their inverses is presented in chapters 10 through 12. More work with analytic geometry is included in chapter 13 as well as transformations and matrices. Space geometry is developed in chapter 14 and chapter 15 treats probability, permutations, and combinations [14, 5].

It is evident that students successfully completing the course, Modern Introductory Analysis, have learned more mathematics than is often presented in many college algebra and trigonometry courses. The material in this course presents a natural development to the study of calculus. The student, who has completed such a study by the end of his junior year, is adequately prepared for the study of calculus.

II. DESCRIPTION OF THE HIGH SCHOOL CALCULUS

Most of the criticisms of teaching high school calculus can be directed to two types of high school courses: (1) a full-year of analytic geometry and calculus which is designed to meet the requirements of the College Entrance Examination Board Advanced Standing in Mathematics; and (2) courses which range in length from only a few weeks to a semester and only introduce limits, differentiation, and integration, and a few limited applications. In the discussion to follow we shall assume that we are speaking of this first type of high school calculus course. A short course of high school calculus is more difficult to justify. More will be said about it later.

In 1960, Albert Blank stated that the kind of high school calculus he favored was an intuitive calculus without a precise "epsilontic" statement of the concept of limit. He wanted the students to obtain clear insights into certain proofs and to be aware that formal proofs are possible and exist even though they themselves would be unable to complete the details of such a proof. It is more important to understand the basic concepts and their applications and to formulate problems in terms of these concepts. "A heightened appreciation of the necessity for precision of statement and logical reasoning can only be obtained for most people after the amassing of considerable detailed experience [7, 539]."

Now Professor Blank is no longer in favor of a rather loosely structured intuitive conceptual calculus. That kind of course was only a beginning. Students and teachers alike want a course equivalent to the best that our

universities are offering. Some teachers who taught calculus successfully for some time found their students having disturbing experiences at some universities. One university would give qualified students advanced placement while another sister university paid no attention to the development level at which the student entered college. Institutions of the second kind gave one reason for its treatment of these students which struck the teachers forcibly: "the conceptual level of high school calculus is too low to justify advanced placement [8, 15]." Thus the teachers will accept nothing less than a first-rate university course.

High school calculus can no longer be a purely formal traditional course. This does not necessarily mean the stressing of complicated "epsilontics" nor that the average student will be asked to supply the detailed punctuation of a proof. It means that the student's thinking is conceptually sound. He may not be able to give a mathematically rigorous proof but he will be able to give a geometrical or analytical interpretation of the basic ideas and notions that lie behind it. "When he does reach the stage of attempting rigorous proof he will appreciate rigor for the sharpness and conviction it brings to knowledge whose values and used he already understands [8, 15]."

III. TEACHER READINESS

It is very wise advice that schools should not introduce calculus until they have reached the appropriate stage of readiness. Arguments are often given that teachers are generally unprepared to teach calculus, that good

textbooks have not been written, and the pre-calculus curriculum is still undecided and has not been standardized. The trouble with waiting until all the conditions are perfect is that one never begins.

The Committee on the Undergraduate Program in Mathematics (CUPM) has listed a number of levels of mathematical preparation which are appropriate for teaching a general curriculum in college mathematics. The first level includes the teaching of the elementary courses at the college level which is appropriate for us to consider. The CUPM recommends that such a teacher should have a strong undergraduate mathematics program which would include the following courses: four semesters of lower division analysis, one semester of lower division probability, one semester of linear algebra, and one semester each of the upper division courses in algebra, analysis, and applied mathematics. The report adds that a stronger major is desirable with options to be selected from courses in probability and statistics, numerical analysis, and differential geometry [13, 6-7].

An effective teacher must maintain an active interest in the communication of ideas and have a dedication to studying, learning, and understanding mathematics at levels significantly beyond those at which he is teaching. The CUPM also states:

> It should be understood that no academic program or degree in itself qualifies an individual to teach effectively at any level unless this preparation is accompanied by a genuine interest in teaching and by professional activities reflecting continuing mathematical growth. These activities may assume the form of several of the following:
> 1) taking additional course work,
> 2) reading and studying to keep aware of new developments and to explore new fields,
> 3) engaging in research for new mathematical results,

4) developing new courses and new ways of teaching,
5) publishing expository or research articles,
6) participating in the activities of professional mathematical organizations [13, 2-3].

The problem of teacher readiness is not one of formal preparation alone, but also a matter of the teachers attitude, mental flexibility, and intellectual honesty. One of the teachers who reviewed a preliminary edition of a calculus text made this comment, "I don't understand all these things, so I guess my students and I will just have to sit down and learn together [8, 14]." This is the type of attitude that teachers must have in order to start teaching a course of this type.

Perhaps you feel the students are going to suffer while this teacher is learning along with her students. The members of the SMSG team disagree, for remember, they are considering only the most select students in the school that are in the advanced placement curriculum. Albert A. Blank, member of SMSG team, writes:

> Given this teacher's attitude, the worst that may happen is that her students will cover less territory than they might. Far worse is the teacher who thinks in rigid formal terms: whose attitude tells the student, "Here are the facts; now memorize them." Is there anything that arouses distaste and deadens curiosity more quickly [8, 14]?

Mr. Eugene Ferguson states that his experience with many excellent traditional mathematics teachers indicates that the teachers have more trouble shifting to the new programs than students do. The reason is that over a period of years teachers have built up certain reactions and language patterns when confronted with a mathematical problem. In the new programs, these problems are often viewed from a different approach, and often the

language and symbolism is changed. To the student there is nothing strange, because this is the first time he has seen it. But the teacher is forced to forget the old approach and learn the new one [21, 145-6].

Mr. Gerald Rising reports that the advanced placement classes he has seen have been excellent. College level texts have been used and the strongest teacher in the department is used. Mr. Rising asks us to compare this with colleges who use inexperienced graduate students overburdened with their own course work. There are, of course, weak teachers at all levels. College students speak of rote teaching in their college classes and the failure of teachers on even the professional level to communicate ideas. Yet, we still maintain faith in college education. A few weak individual teachers cannot keep us from making curricula advances [29, 288].

IV. MEANS OF ACCELERATION

In order to have time for calculus in high school, some means of acceleration is necessary. Several methods ofacceleration have been used. One method is accomplished by offering two courses, usually geometry and algebra II, to be taken in the same year. A second method of acceleration, often used in three-year high schools, is to group the students into special honors programs in which the three-year mathematics sequence is completed in two years. The most favored means is to eliminate wasted time in the seventh and eighth grades by setting up a new mathematics program which would allow elementary algebra to be taken in the eighth grade by the students in the advanced placement program. Thus the preparation for the calculus

could be accomplished by the twelfth grade for these students. All this leads to the question of whether calculus is really the course to offer those students who are energetic enough to elect another year of mathematics in the twelfth grade.

For the advocate of calculus in high school, the answer is "yes, offer a full year of calculus and analytic geometry in the twelfth grade." W. E. Ferguson, who has taught calculus in colleges from 1940 to 1954, states:

> At various times I was asked by high school teachers, "In the senior year, after we have finished solid geometry and trigonometry, should we start teaching the calculus?" In the brief moment after the question was asked all I could see was that miserable set of papers from my differential calculus class that showed that they really were weak in algebraic manipulation, so my answer became a standard one: "No, don't teach calculus, teach them more algebra and how to carry through an algebraic manipulation involving complex fractions and fractional exponents without making errors."[20, 451-2]

Mr. Ferguson goes on to say that since 1960, the omission of teaching calculus in his high school would be unthinkable. However, before a high school offers calculus, he stresses strong conditions that first must be met. These conditions are as follows:

1) The school must have a program in mathematics which allows the student to complete four years of high school mathematics, similar to the courses previously described, by the end of his junior year.
2) There must be a teacher on the staff capable of teaching a calculus course on the college level as outlined in the Advanced Placement Program of the College Entrance Examination Board.
3) The student must be adequately prepared mathematically, properly motivated, and willing to spend eight to ten hours a week on homework [20, 451-2].

V. STUDENTS CAN LEARN MORE MATHEMATICS

The question we should consider now is, "Are students really capable of learning more than they already are?" Mr. Irving Adler reports the mathematical ability of children in grades K-12 has been underestimated for the bright students as well as for the average student. Even the student who is retarded by our standards of today can be taught more sophisticated mathematics than previously thought possible. What students now learn depends to a great extent on the methods we use in teaching and what we try to teach them. By making use of educational psychology and reorganizing the traditional mathematics and integrating it with the new ideas of the modern mathematics the general level of mathematics learned by students can be raised considerably [1, 212].

Mr. Adler refers to the theories of learning proposed by Jean Pioget, a Swiss psychologist who has spent many years studying how children acquire mathematical ideas. Her theories state that the ages for the stages in the development of a child's thinking is not constant and can be lowered, thus making it possible to teach more advanced mathematical concepts at a lower age.

The critics of high school calculus indicate that high school students only learn the mechanical applications of formulas without understanding. According to Gerald R. Rising, Norwalk Public Schools, Norwalk, Connecticut, students who have successfully completed this first type of calculus course described and passed the CEEB examination, know much more than

just formulas. Students who have scored well on the CEEB examination do as well as other control groups of college students [29, 287-90].

The study by William D. McKillip, "The Effects of High School Calculus on Students' First-Semester Calculus Grades at the University of Virginia [25, 472]," indicates that a high school calculus course of two or more semesters does contribute significantly to improving student's grades in the first semester of college calculus, when advanced placement was not given.

Mr. Rising says that the answer to our high-quality students is certainly not more of the same work they have already done. This is too close to the old philosophy in which bright youngsters were simply assigned a dozen more problems than the average student. In order for students to remain in the advanced placement programs they need to know their basic mathematics well, and they do. He reports that from his experience with these students he would gladly compare them with second-semester college freshmen.

VI. THE SHORT HIGH SCHOOL CALCULUS COURSE

Although a short introduction to calculus in a semester or less is difficult to defend, Mr. Gerald R. Rising does believe this type of unit can have real merit in the articulation of students from high school mathematics to college mathematics. There are two topics, limits and the definition of the derivative, which most college professors will agree deserve more time than they usually give them [29, 287-90].

The secondary school program offers the possibility of spending

several weeks on each topic whereas colleges often skim over these topics superfically in only several class periods. Mr. Rising believes that it is not necessary to teach any formulas in this type of a short calculus course. The definition of the derivative would be the only means of differentiation. These students should certainly not be allowed to skip a semester of calculus but by the rigorous teaching of these topics the high school teacher will be helping his students through two of the toughest hurdles in the college mathematics program.

CHAPTER IV

THE CASE AGAINST CALCULUS IN HIGH SCHOOL

Recent years have witnessed an increasing concern over the problems of grade placement and instructional procedures as they are related to calculus. Preparation for calculus has been one of the principal objectives of the reform movement in mathematical education. Along with this there has been considerable speculation about the desirability of calculus as a high school subject. With the rapid rate of increase in the number of students taking advanced placement mathematics it is time to examine the high school calculus problem for several reasons: (1) in many high schools calculus is commonly being taught to students; (2) the nature of the calculus course has a major impact on the nature of the pre-calculus courses taught; (3) what is happening in the advanced placement mathematics may not be in the best interests of all the students enrolled in these courses [3, 482-3].

The main concern of mathematics educators is for the best interests of the superior students, so that the student's articulation to college may be made with ease and that they will be able to proceed at a rapid pace, with as much mathematical background as their ability permits.

I. SHOULD CALCULUS BE PART OF THE HIGH SCHOOL PROGRAM?

The opinions of college mathematics teachers in general are not complementary regarding the background and ability of the advanced placement students. Dr. J. H. Neelley, of Carnegie Institute of Technology,

complains about some of the freshmen who had calculus in high school knowing merely some formulas and not any of the "whys" of calculus [26, 584-6]. The following quotation was found to be typical of many responses.

> In the classroom we find that often the students look forward to that part of the semester where we will cover material which they had in high school. They live in the expectation that it will be then that they will show their sterling qualities. Unfortunately, all too often, their performance at this point corresponds to their performance in the other areas of that course. As far as we can determine, the high school work often is drill work, which is more or less learned by rote, whereas we demand more. The pace at which we operate is different and we often spend days on material to which weeks are devoted in high school [22, 561].

In a survey of the opinions of Ohio college mathematics department heads concerning the question relative to the inclusion of the calculus in the high school curriculum, Mr. Robert S. Brown reported that 69 percent of those responding to his questionare felt that calculus should not be a part of the bigh school program [9, 245-7]. Twenty-five percent were in favor and 6 percent answered "maybe". Strong qualifying statements were written by most of those answering "yes" or "maybe". Typical statements were that calculus should be taught "only if it is taught as a full-year, college-level course, by a college-caliber teacher, to a very select group for advanced placement purposes [9, 246]." Those answering "no" expressed the opinion that high school students were not mature enough to appreciate many of the concepts of calculus; that too many other worthwhile topics of mathematics would have to be sacrificed; that high schools should offer a good pre-calculus course, but that the teaching of calculus should be left to the colleges; that

calculus without a good foundation in analytic geometry is only a delusion; and that the mechanics of calculus are learned but it is accompanied by very little understanding of the concepts [9, 246].

II. THE EFFECT OF HIGH SCHOOL CALCULUS ON THE STUDENT

Is the performance in the first semester of college calculus affected by the calculus course taken by the student in high school? In the study at the University of Virginia, William D. McKillip concludes that the grades of those who had taken at least one semester but less than two complete semesters of high school calculus were not significantly better than the grades they would have been expected to receive without the previous calculus [25, 470-2]. However, the study done by Donald Tillotson at the University of Kansas indicated no evidence that having studied a unit on calculus in the twelfth grade was actually harmful to students' subsequent performance in college calculus.

From these studies one concludes that a one-semester course in calculus would have to justify the course on grounds other than the improvement of success in college calculus. On the basis of a government survey, Lauren G. Woodby recommends that courses in calculus less than a full year in length should not be offered. He believes if calculus is to be offered in high schools it should be a full-year course comparable to the college-level courses taught in the universities.

III. THE IMPORTANCE OF CALCULUS

One argument for early calculus is that calculus is so overwhelmingly important that it should displace any other subject in the curriculum. Mr. Carl B. Allendoerfer asks, "But for what is it so important [3, 483]?" The physicists, chemists, and engineers say that calculus is a necessary prerequisite for their courses, but according to Mr. Allendoerfer they later admit that even more important than calculus is a knowledge of algebra, geometry, and trigonometry. He advises that before you upset your curriculum teaching calculus you should look at the use made of it in these sciences and engineering courses. He is convinced that although calculus is an essential part of a mathematical education, the case for its teaching has been over emphasized in the relation to other subjects.

Occasionally schools have been put under political pressure from the public to modernize their mathematics teaching, which is good. But too often this can result in a crash program in which very little time is given for the developing of a sound course of action. This may result in a course's being taught by inadequately prepared teachers to students who are superior but who are not yet ready for the calculus. The course could become a meaningless mechanical manipulation of symbols that results in inadequate preparation of an area of mathematics very essential to the understanding of further advanced mathematics. When students have to repeat their work in college calculus, due to this kind of background, the outcome is often the well-known problems associated with such a repetition [6, 29].

The challenge of taking a new course is gone when a student repeats a course. A student's previous exposure to a course may give him a feeling of false security which may develop poor study habits. This student could also have a demoralizing effect on his college classmates who are struggling for the first time with the sections of calculus which are difficult to understand. The college teacher may also receive criticism from this student if he wishes to develop his course differently from the method used by the student's high school teacher. Mr. Grossman says,

> Why stultify college freshman mathematics by fixing it for our superior high school students? Why give the colleges the difficult public relations job of pacifying these students coming to colleges where advanced placement mathematics cuts across college courses, meaning that, if students omit any course, they miss important sets of units? [22, 562]

IV. MAGIC IN THE WORD "CALCULUS"

The magic of the name "calculus" is often a strong motivating factor for high school students to take calculus. It is a name associated with college and has "enrichment" written all over it. It seems to be a prestige factor for high school students to say they are taking calculus and a way of showing off to their less elite schoolmates. They feel like they are already in college while still high school students. Unfortunately this attitude is not limited only to students. School systems often are also put under political pressure to show off because other neighboring schools may be offering such a program.

V. THE PURPOSE OF ADVANCED PLACEMENT MATHEMATICS

In the discussions concerning advanced placement mathematics, mathematics educators have often been talking about two different objectives, enrichment and acceleration. By enrichment we mean the teaching the best mathematics that the students' abilities permit. A special emphasis is placed upon understanding and a student's ability to think and reason. By acceleration we mean the teaching of as much sequential mathematics as possible so that the students can more rapidly move into advanced areas of college mathematics.

Mr. Grossman states that the original purpose of the advanced placement program was acceleration. To accelerate would mean a student could complete his freshman year of college mathematics while in high school. He would then be able to take more advanced mathematics the first year of college, enabling him to reach honors mathematics or graduate mathematics much sooner. For some students this means having the calculus as a tool for college courses in engineering, physics, and other sciences. For these students, mature enough to accelerate and ready for calculus, who will be going to colleges that accept their high school credits and have a program into which they readily fit, the high school calculus may be satisfactory. However, he feels that for the superior student enrichment is a better objective than acceleration.

According to Mr. George Grossman, the original objectives of the advanced placement mathematics has been broadened. He states that there

are many students in advanced placement mathematics who are not planning to major in mathematics, engineering, or science. They have been encouraged to take this program with the sole objective of completing in high school their one year of required college mathematics--to save college tuition; to avoid having to take mathematics in college; or to leave room in their college programs for courses in their major fields of interest. This is not to imply that these objectives in themselves are completely bad.

VI. THE HIGH SCHOOL CALCULUS TEACHER

Calculus is a difficult subject to teach well and should be taught only by the best trained and experienced teacher. The argument is often given by many high school teachers that in high school, advanced placement courses are taught by superior teachers, whereas often in many colleges freshmen and sophomores are taught by graduate students with little experience and often problems of their own which results in very poor teaching. Is this a reason for teaching calculus in high school? We should rather get the colleges to consider the quality of instruction given to freshmen and sophomores.

Mr. George Grossman feels that many of the high school calculus courses now given are "once over lightly" courses with a minimum of rigor and understanding because the teacher does not have the necessary mathematical maturity and knowledge of needed advanced mathematics. This may be one of the reasons for many of the complaints from our colleges. To teach the calculus properly, the teacher must know advanced calculus and a good deal of the theory of functions. He says that, "The first problem a

school must solve when planning for its superior students is to get superior teachers." "The second problem is what these teachers are to teach [22, 564]."

During the academic year 1965-1966, Dr. W. M. Perel, Professor of Mathematics at Wichita State University, directed an In-service Institute for secondary mathematics teachers. The institute was held at the University of North Carolina and was supported by the National Science Foundation. Of the 64 applicants, all of whom were mathematics teachers, only a little more than half, 61 percent, had undergraduate majors in mathematics and only one out of 11 teachers with graduate degrees had his graduate degree in mathematics. Even these figures are misleading, for 28 percent of the applicants had no calculus, 44 percent had no course work beyond calculus, and only 25 percent had more than two courses above calculus. A teacher with no more than two courses above calculus, which does not even satisfy current certification requirements, can hardly be said to have a bona fide major in mathematics [27, 289].

Mr. C. C. MacDuffee states that the main reason why many high schools do not favor teaching analytic geometry or calculus is that their teachers feel incapable of teaching it. Schools, failing to acquire mathematics teachers, gave mathematics classes to football coaches, social science teachers, and the like on an emergency basis long ago, and the emergency became permanent. In Minnesota approximately half of the high school mathematics teachers were certified to teach the subject. Similar situations exist in other states. It is understandable that teachers not well

prepared in their subject are not eager to introduce more advanced courses [24, 4].

The University of Wiscon-in gives degrees in education with a major in mathematics and a teacher's certificate to persons who have completed: a year of calculus, and a semester each of the theory of equations, college geometry, projective geometry, and differential equations, or an equivalent program. Mr. MacDuffee believes that these teachers are quite capable of instructing a high school calculus class [24, 3-4].

VII. THE SHORT CALCULUS COURSE

Mr. Allendoerfer favors a short course, four to six weeks, in calculus at the end of the twelfth grade as a good transition to the freshman year in college. He is of course opposed to a longer course but less than a full year because it duplicates what a student will again take in college. This wastes the pupil's time and spoils his appetite for college calculus. The Commission on Mathematics of the CEEB discourages the teaching of a short formal calculus course because "it tends to breed over confidence and blunt the exciting impact of a thorough preparation [22, 563]."

In answer to the question: "If we do not offer the calculus in high school, do you think it advisable to include a short introduction to calculus during the senior year?," Robert S. Brown stated that 6 percent of the college department heads again answered "maybe" and 25 percent answered "yes" stating that it could do no harm. The 69 percent, of the college department heads, that answered "no" expressed themselves more vehemently

than the others. The following quotes are typical of the responses that Mr. Brown received.

> 1.) My experience has been that freshmen who have had a brief introduction to calculus often have a false sense of security that produces more troubles than might have developed had they started from scratch.
>
> 2.) An introduction to calculus for less than an entire year serves only to dull the student's appetite for mathematical analysis.
>
> 3.) My experience has been that such courses tend to skim off the interesting cream and leave the student thinking he knows more than he really does; so he underestimates his fellow-classmates' ability and tends to be bored with a course that contains partly material he has had and partly material that is new.
>
> 4.) It creates only false security in the student and often gives him misinformation.
>
> 5.) Better no calculus than just a little.
>
> 6.) Such students do not appreciate rigor and careful proof.
>
> 7.) There are more important topics to be covered.
>
> 8.) I have had a number of students who have been introduced to calculus; they all have a mistaken notion as to their ability to understand basic concepts and techniques [9, 264].

Mr. Grossman, also, feels we should rethink the CEEB's objection to a shorter course in the calculus. He believes that if units of calculus are properly developed they can be made part of high school courses just as units in analytic geometry and sometimes trigonometry are spread throughout the four years of high school. If the same type of development for the calculus in high school could be given we may find that students are more prepared to receive these newer concepts.

CHAPTER V

SUMMARY OF THE CASES: PRO AND CON

Arguments for and against the high school calculus generally deal with one of two questions, "Can we teach calculus in the high school?" and "Should we teach calculus in the high school?" The feasibility of offering such a course generally centers around the capabilities of the teacher, the difficulty of the subject, and the maturity of the pupils, The advisability of such instruction is most often discussed on the bases of value for general education, value as background for future study, the relative value as compared with other topics which might be studied, and the effect of such study on a student's attitudes toward later instruction in the calculus [31, 37].

I. GENERAL EDUCATION VALUE

Many of the early advocates of high school calculus emphasized its value in a general education saying that calculus was of value to students not going on to college as well as to those who were. It would be of value to these students when reading scientific articles of a popular nature. The American civilization is becoming more technical and the principles of calculus are valuable background for the well-informed citizen [31, 37-8].

II. VALUE AS BACKGROUND FOR FUTURE STUDY

Mr. Noah Bryan Rosenberger says those who are familiar with courses in engineering, physics, and mechanics, know that calculus furnishes many of

the fundamental principles in these fields. Calculus is one of the most important aids in applied mathematics and contributes largely to the field of pure mathematics [30, 154]. Not only does calculus provide an indespensable background for students of science but also for students of social studies and business.

The important position occupied by calculus in the mathematics structure gives wide knowledge and experience upon which a formal mathematical system can be built. Thus says Mr. Rosenberger, high school seniors ready for calculus and who are interested in mathematics should be given the opportunity to become acquainted with the subject.

III. DIFFICULTY OF CALCULUS

Mr. Tillotson states that the arguments concerning the difficulty of calculus for high school students have been based on personal opinion with very little substantiation. He indicates that studies by Lehi Smith show that junior high school students can acquire the concept of a limit. This is where a good background in mathematics is important, for experience rather than maturity seems to be an important factor in understanding this concept [30, 42].

Mr. Howard F. Fehr advocates calculus for high school students because he says then they will be able to learn the theory of calculus much better when taken in college. He writes:

> If students first hear of limits in their second year of college upon taking up the study of the calculus, they do as most students of the past have done, learn the tricks of differentiating and integrating without knowing what it is they are doing. In fact the

> pupil is so busy learning and applying the new symbolism and formulae, that he has no time left to study the underlying theory of limits and theorems of Rolle, the Mean, Taylor and others [19, 298].

Some critics of high school calculus say that such a course may result in merely manipulative facility without basic understanding but this is a constant threat at whatever level the course is offered. The same line of reasoning could be used against algebra or trigonometry. Test results from the College Entrance Examination Board indicate that students who have completed a rigorous high school calculus course under a qualified teacher are gaining a good understanding of the concepts of calculus.

The study by William McKillip also supports the opinion that calculus can be taught successfully in high school. He found that the grades in the first semester of calculus of students who had taken two or more semesters of calculus in high school were significantly better than the grades they would have been expected to earn had they not taken calculus in high school.

IV. THE CAPABILITIES OF THE TEACHER

The availability of qualified teachers is one factor that has affected both the possibility and advisability of expanding the high school curriculum to include calculus. Mr. Tillotson states that some educators of mathematics consider several courses in analysis beyond a course in calculus to be sufficient background for a high school calculus teacher. As was noted before, the CUPM stresses the importance of a much stronger background, one that would include courses in theoretical analysis and the theory of real and complex variables.

Although many secondary teachers consider their background in analysis to be very meager it is greater than that in statistics or modern algebra. This may explain the number of schools attempting to introduce calculus rather than other advanced topics of mathematics. Mr. Tillotson supports this by saying of the high school mathematics teachers in Kansas in 1957 to 1958, 41.8 percent had credit for analytic geometry and calculus, 22.6 percent had taken some additional hours in analysis, but only 11.8 percent had any abstract algebra, and 16.4 percent had credit in probability and statistics [31, 45].

V. CONDITIONS NECESSARY BEFORE TEACHING HIGH SCHOOL CALCULUS

"The individual teacher is the most important factor in the development of a strong mathematics program [32, 34]." In-service training programs such as the National Science Foundations Institutes have been influential in preparing teachers to teach the emerging twelfth-grade courses. Lauren G. Woodby concludes from a government survey that college-level calculus courses can be successfully taught at the high school level provided the teacher is adequately prepared to teach calculus and that there are enough capable students with the proper mathematical background (as described in chapter 3) and a desire to learn calculus.

VI. CONTENT OF THE FOURTH-YEAR HIGH SCHOOL MATHEMATICS COURSE

There is still lack of agreement on the mathematics that should be taught after algebra II for the college-bound students. Many different courses have been proposed and many different ones are being taught, but according to Mr. Woodby no one particular program seems to be the most appropriate at the present time.

Mr. Woodby states that there is little acceptance of a course in probability and statistics as the fourth or fifth-year mathematics course in the college-preparatory program. There is even less acceptance of courses in linear algebra, matrices, and computer mathematics. This situation is probably due to the lack of training in these areas by the majority of high school teachers [32, 34-6].

In the survey conducted by O. Lexton Buchanan, Jr., regarding the opinions of college teachers of mathematics concerning the twelfth-year course, analytic geometry was the most popular choice for the second semester course. Additional elementary functions was the second most popular followed by probability and statistics, third; and matrix algebra fourth. Each of these courses was selected by at least 40 percent of the respondents as either a first or second choice. Modern algebra, calculus, and a combination course, ranked in that order, received no more than 23 percent of the vote. The survey showed that teaching a unit on limits in the twelfth-grade was not as controversial as teaching a unit of calculus among the college teachers of mathematics included in this survey [10, 223-5] .

From a survey of the literature it appears that there is no set fourth or fifth-year mathematics course that will be suitable for all high schools to offer their college-bound students. The mathematics curriculum of a particular school needs to be determined by the faculty of that school after they have carefully studied and analyzed all the factors involved. However, a fourth-year course such as the one offered by Houghton Mifflin appears to be quite satisfactory as either a terminal mathematics course or as a pre-calculus course.

Small schools with very few students desiring to take advanced mathematics have more of a problem due to the feasability of offering a fourth and fifth-year mathematics course. Mr. Woodby suggests a good collection of mathematics materials be provided for the library as the most effective solution for small high schools [32, 36].

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THE CONTROVERSY OF TEACHING CALCULUS IN HIGH SCHOOL

by

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The main purpose of this report is to summarize the literature concerning the controversy of teaching calculus in high school. This will enable teachers and administrators to (1) gain insight into the problems they face in their school, (2) determine under what conditions calculus can be taught successfully in high school, (3) analyze the framework under which students may take calculus in high school, and (4) aid them in determining the content of the fourth year mathematics course for their high school curriculum.

Calculus has been taught in a few high schools in the United States since the middle of the nineteenth century. From this early beginning to the present there have been educators who have tried to establish the te ching of calculus in high school with varying degrees of success. Between the turn of the century and the end of World War II, most high schools ceased teaching calculus. This seems to be due to a re-evaluation of the role of the high school and the lack of adequate textbooks and qualified teachers.

In the mid-fifties, the thought of teaching calculus in high school again became popular among many educators. The advocates of high school calculus stressed its importance for a general education in a technically oriented nation and its importance as background for those entering scientific fields. Many educators also caution high schools of offering calculus before they are ready, warning them of the inherent dangers. There is the possibility of the student's acquiring incorrect or over-simplified concepts, of his learning mechanical manipulation with very little understanding of the concepts, and of his over-estimating his knowledge of

calculus thus not exerting enough effort in college calculus. In contrast the student can be introduced to basic concepts which may not be readily acquired in the more rapid pace of the college course. Teachers need to be aware of the positive and negative effects of high school calculus and teach accordingly.

Before high schools offer calculus they need to have a teacher with a strong background in analysis and enough students with a desire to learn calculus. These students should successfully complete four years of high school mathematics by the end of their junior year. The course should be a full year in length and taught on a college level. Test results from the CEEB and individual surveys indicate that calculus can be taught successfully with understanding to high school students under conditions such as these.

There does not seem to be any single fourth-year high school mathematics or pre-calculus course most appropriate for the college-bound student. Surveys indicate that analytic geometry is the most popular choice for this course followed by elementary functions, and probability and statistics.